HOMEWORK 11 – ANSWERS TO (MOST) PROBLEMS

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Section 5.3: The Fundamental Theorem of Calculus

5.3.43. $1 + (-1) = 0$ (split up the integral into $\int_0^{\frac{\pi}{2}} \sin(x)dx + \int_{\frac{\pi}{2}}^{\pi} \cos(x)dx$)

5.3.45. $\frac{1}{x^4}$ is discontinuous at 0 (the FTC applies only to continuous functions)

5.3.57. $F'(x) = 2xe^{x^4} - e^{x^2}$

5.3.67.

(a) $g'(x) = f(x) = 0 \Rightarrow x = 1, 3, 5, 7, 9$, but 9 is an endpoints, so ignore it. Hence, by the second derivative test:

- $g''(1) = f'(1) < 0$, so g has a local max at 1
- $g''(3) = f'(3) > 0$, so g has a local min at 3
- $g''(5) = f'(5) < 0$, so g has a local max at 5
- $g''(7) = f'(7) > 0$, so g has a local min at 7

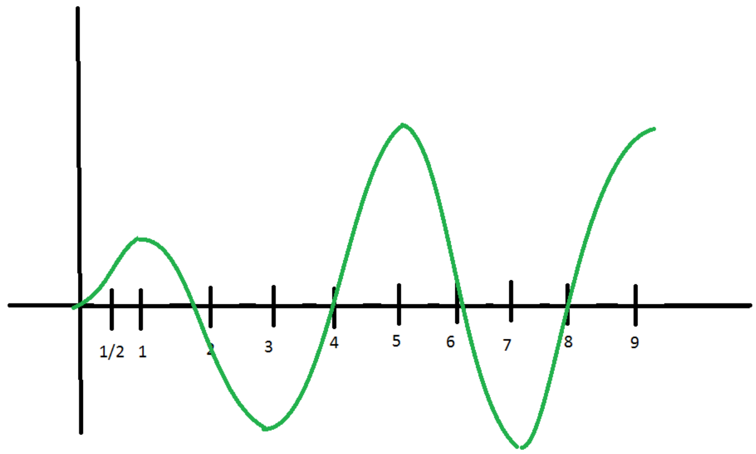
In summary, g attains a local minimum at $\boxed{3 \text{ and } 7}$, and a local maximum at $\boxed{1 \text{ and } 5}$.

(b) You do this by guessing. The candidates are $0, 1, 3, 5, 7, 9$ (critical points and endpoints). Notice $g(0) = 0, g(3) < 0$ but $g(5) > 0$, so you can eliminate 0 and 3. Also $g(5) > g(1)$, so you can eliminate 1. Also $g(7) < 0$, so you can eliminate 7. This leaves us with 5 and 9, but notice that $g(5) = g(9)$ (the areas between 5 and 9 cancel out), so the answer is $\boxed{x = 5 \text{ and } x = 9}$ (the book only writes $x = 9$, but I disagree)

(c) $g''(x) = f'(x)$, so to see where g is concave down, we have to check where $f'(x) < 0$, i.e. where f is decreasing. The answer is $\boxed{\left(\frac{1}{2}, 2\right) \cup (4, 6) \cup (8, 9)}$.

Date: Monday, December 2nd, 2013.

(d) 1A/Math 1A - Fall 2013/Homeworks/FTCSol.png



5.3.70. First rewrite the limit as:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\sqrt{\frac{i}{n}}$$

And you should recognize that $\Delta x = \frac{1}{n}$, $f(x) = \sqrt{x}$, $x_i = \frac{i}{n}$. In particular $a = x_0 = 0$ and $b = x_n = \frac{n}{n} = 1$, so in fact this limit equals to:

$$\int_0^1 \sqrt{x}dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$$

Section 5.4: Indefinite Integrals and the Net Change Theorem

5.4.10. $\frac{1}{6}v^6 + v^4 + 2v^2 + C$ (expand out)

5.4.12. $\frac{x^3}{3} + x + \tan^{-1}(x) + C$

5.4.25. -2 (expand out)

5.4.31. $\frac{55}{63}$ (Write this as $x^{\frac{4}{3}} + x^{\frac{5}{4}}$, with antiderivative $\frac{3}{7}x^{\frac{7}{3}} + \frac{4}{9}x^{\frac{9}{4}}$)

5.4.37. $1 + \frac{\pi}{4}$ (Antiderivative is $\tan(\theta) + \theta$, because:)

$$\frac{1+\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \sec^2(\theta) + 1$$

5.4.49. $\frac{4}{3}$ (antiderivative is $y^2 - \frac{y^3}{3}$)

5.4.54. The bee population after 15 weeks

5.4.62.

(a)

$$s(6)-s(1)=\int_1^6 v(t)dt=\int_1^6\left(t^2-2t-8\right)dt=\left[\frac{t^3}{3}-t^2-8t\right]_1^6=-12+\frac{26}{3}=-\frac{10}{3}$$

(Alternatively, you could have just calculated $s(t)$ by antidifferentiating v and then calculated $s(6)-s(1)$ directly)

(b) Notice that $v(t)=(t+2)(t-4)=0$, which gives $t=4$ (since $t\geq 0$). So in particular $v(t)\leq 0$ on $[1,4]$ (the particle is moving to the left) and $v(t)\geq 0$ on $[4,6]$ (the particle is moving to the right), hence we must find:

$$\begin{aligned}s(1)-s(4)+s(6)-s(4)&=-\left(s(4)-s(1)\right)+\left(s(6)-s(4)\right)\\&=-\int_1^4 v(t)dt+\int_2^6 v(t)dt\\&=-\int_1^4\left(t^2-2t-8\right)dt+\int_4^6\left(t^2-2t-8\right)dt\\&=-\left[\frac{t^3}{3}-t^2-8t\right]_1^4+\left[\frac{t^3}{3}-t^2-8t\right]_4^6\\&=-(-18)+\frac{44}{3}\\&=\frac{98}{3}\end{aligned}$$

5.4.64. 1800 (antiderivative is $200t-2t^2$, $a=0$, $b=10$)

Section 5.5: The substitution rule

5.5.7. $\frac{1}{2}\cos(x^2)+C$ ($u=x^2$, $du=2xdx$)

5.5.31. $e^{\tan(x)}+C$ ($u=\tan(x)$, $du=\sec^2(x)dx$)

5.5.33. $-\frac{1}{\sin(x)}$ ($u=\sin(x)$, $du=\cos(x)dx$)

5.5.48. $\frac{1}{5}(x^2+1)^{\frac{5}{2}}-\frac{1}{3}(x^2+1)^{\frac{3}{2}}$ ($u=x^2+1$, $du=2xdx$, $x^2=u-1$)

5.5.59. $e-\sqrt{e}$ ($u=\frac{1}{x}$, $du=-\frac{1}{x^2}dx$, $a=1$, $b=\frac{1}{2}$)

5.5.62. $\sin(1)$ ($u=\sin(x)$, $du=\cos(x)$, $a=0$, $b=1$)

5.5.77. $0+6\pi$ (the first integral is 0 because the function is an odd function, or use $u=4-x^2$, $du=-2xdx$, $a=0$, $b=0$, and the second integral represents the area of a semicircle with radius 2)

5.5.86. Using the substitution $u = x^2$, we get $du = 2xdx$, so $xdx = \frac{1}{2}du$. Moreover, the endpoints become $u(0) = 0$ and $u(3) = 9$, so:

$$\int_0^3 xf\left(x^2\right) dx = \int_0^9 f(u)\frac{1}{2}du = \frac{1}{2}\int_0^9 f(x)dx = \frac{4}{2} = 2$$

5.5.92.

(a) For the first integral, let $u = \cos(x)$, then $du = -\sin(x)dx = -\sqrt{1-u^2}dx$, so the first integral becomes $\int_1^0 \frac{f(u)}{-\sqrt{1-u^2}}du = \int_0^1 \frac{f(u)}{\sqrt{1-u^2}}du$. For the second integral, let $u = \sin(x)$, then $du = \cos(x)dx = \sqrt{1-u^2}dx$, so the second integral becomes $\int_0^1 \frac{f(u)}{\sqrt{1-u^2}}du$, and it is now clear that both integrals are equal!

(b) By (a) with $f(x) = x^2$ (for the first step), and the fact that $\sin^2(x) = 1 - \cos^2(x)$, we get:

$$\int_0^{\frac{\pi}{2}} \cos^2(x)dx = \int_0^{\frac{\pi}{2}} \sin^2(x)dx = \int_0^{\frac{\pi}{2}} 1dx - \int_0^{\frac{\pi}{2}} \cos^2(x)dx = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos^2(x)dx$$

Solving for $\int_0^2 \cos^2(x)dx$, we get: $\boxed{\int_0^2 \cos^2(x)dx = \frac{\pi}{4}}$, and hence $\boxed{\int_0^2 \sin^2(x)dx = \frac{\pi}{4}}$ (by (a))